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INTEGRALS PERTAINING TO I-FUNCTIONS

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ABSTRACT

In this paper, certain integrals involving product of the I- function and Fox - Wright's Generalized Hypergeometric Function have been established. Being unified and general in nature, these integrals yield a number of known and new results as particular cases. For the sake of illustration, some special cases are also recorded here.

KEYWORDS: Fox - Wright's Generalized Hypergeometric Function, I-function

AMS Subject Classification: 33C45, 33C60

1. INTRODUCTION

In the present paper, we have used the I-function given by Saxena [1], defined as:

$$I^{m,n}_{p_i,q_i:r}[z] = I^{m,n}_{p_i,q_i:r}\left[z \left| \begin{matrix} (a_{j'},\alpha_{j'})_{1,n}; (a_{j'i'},\alpha_{j'i'})_{n+1,p_i} \\ (b_{j'},\beta_{j'})_{1,m}; (b_{j'i'},\beta_{j'i'})_{m+1,q_i} \end{matrix} \right. \right]$$

$$= \frac{1}{2\pi\omega}\int_L \varphi(\xi)\, z^{\xi}\, d\xi \quad , \tag{1}$$

Where

$$\varphi(\xi) = \frac{\prod_{j'=1}^{m}\Gamma(b_{j'}-\beta_{j'}\xi)\prod_{j'=1}^{n}\Gamma(1-a_{j'}+\alpha_{j'}\xi)}{\sum_{i'=1}^{r}\left\{\prod_{j'=m+1}^{q_i}\Gamma(1-b_{j'i'}-\beta_{j'i'}\xi)\prod_{j'=n+1}^{p_i}\Gamma(a_{j'i'}-\alpha_{j'i'}\xi)\right\}} \tag{2}$$

and $\omega=\sqrt{-1}$. For the conditions on the several parameters of the I-function, one can refer to [1].

Also, the Wright's Generalized hypergeometric function ${}_p\psi_q$ ([2], also see [3]) appearing in this paper is defined as:

$${}_p\psi_q\left[\begin{matrix} (e_1,\gamma_1),\ldots,(e_p,\gamma_p); \\ (f_1,\delta_1),\ldots,(f_q,\delta_q); \end{matrix} x \right] = {}_p\psi_q\left[\begin{matrix} (e_j,\gamma_j)_{1,p}; \\ (f_j,\delta_j)_{1,q}; \end{matrix} x \right] = \sum_{k=0}^{\infty} \frac{\prod_{j=1}^{p}\Gamma(e_j+\gamma_j k)}{\prod_{j=1}^{q}\Gamma(f_j+\delta_j k)}\frac{x^k}{k!} \tag{3}$$

where γ_i and δ_j (i = 1, …, p ; j = 1, …, q) are real and positive, and $1 + \sum_{j=1}^{q}\delta_j - \sum_{j=1}^{p}\gamma_j > 0$.

To establish the integrals we have also used the following result due to Rainville [4]:

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} A(k,n-k) \tag{4}$$

2. MAIN RESULTS

In this section, we have evaluated certain integrals involving product of the I - function, Wright's Generalized hypergeometric function and exponential function.

First Integral

$$I_1 \equiv \int_0^t x^{\rho-1}(t-x)^{\sigma-1} e^{-xz} {}_p\psi_q\left[\begin{matrix}(e_j,\gamma_j)_{1,p};\\(f_j,\delta_j)_{1,q};\end{matrix}\, ax^{\zeta}(t-x)^{\eta}\right]$$

$$\times\, I^{m,n}_{p_i,q_i:r}\left[yx^{\mu}(t-x)^{\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_j,\alpha_j)_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right] dx$$

$$= e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{u} f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times\, I^{m,n+2}_{p_i+2,q_i+1:r}\left\{yt^{\mu+\nu}\left|\begin{matrix}(1-\rho-\zeta k,\mu),(1-\sigma-(n-1)k-u,\nu)(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i},(1-\rho-\sigma-(\zeta+\eta-1)k-u,\mu+\nu)\end{matrix}\right.\right\} \tag{5}$$

Where
$$f(k) = \frac{\prod_{j=1}^{p}\Gamma(e_j+\gamma_j k)}{\prod_{j=1}^{q}\Gamma(f_j+\delta_j k)}\frac{a^k}{k!} \tag{6}$$

provided $(i)\ \mu \geq 0, \nu \geq 0$ (Not both zero simultaneously)

(ii) ζ and η are non-negative integers such that $\zeta+\eta \geq 1$

(iii) $A_i > 0$, $B_i < 0$; $|\arg y| < \frac{1}{2} A_i\pi$, $\forall\ i \in 1,\ldots,r;$

where
$$A_i = \sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p_i}\alpha_{ji} + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q_i}\beta_{ji},$$

$$B_i = \frac{1}{2}(p_i - q_i) + \sum_{j=1}^{q_i} b_{ji} - \sum_{j=1}^{p_i} a_{ji},$$

and

(iv) $\operatorname{Re}(\rho) + \mu \min_{1\leq j\leq m}\left[\operatorname{Re}\left(b_j/\beta_j\right)\right] > 0$, $\quad \operatorname{Re}(\sigma) + \nu \min_{1\leq j\leq m}\left[\operatorname{Re}\left(b_j/\beta_j\right)\right] > 0.$

Proof

$$I_1 = e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1} e^{(t-x)z} \; {}_p\psi_q\left[\begin{matrix}(e_j,\gamma_j)_{1,p};\\(f_j,\delta_j)_{1,q};\end{matrix}\; ax^{\zeta}(t-x)^{\eta}\right]$$

$$\times\; I^{m,n}_{p_i,q_i;r}\left[y\,x^{\mu}(t-x)^{\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_j,\alpha_j)_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right] dx$$

Now replacing $e^{(t-x)z}$ by $\sum_{u=0}^{\infty}\frac{(t-x)^u}{u!}z^u$ and also using the expressions (1) and (3), we get

$$I_1 = e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\sum_{u=0}^{\infty}\frac{(t-x)^u}{u!}z^u\sum_{k=0}^{\infty}\frac{\prod_{j=1}^{p}\Gamma(e_j+\gamma_j k)}{\prod_{j=1}^{q}\Gamma(f_j+\delta_j k)}\frac{a^k x^{\zeta k}(t-x)^{\eta k}}{k!}$$

$$\times\frac{1}{2\pi i}\int_L \varphi(\xi)\,y^{\xi}x^{\mu\xi}(t-x)^{\nu\xi}\,d\xi\,dx$$

$$= e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{\infty}\frac{\prod_{j=1}^{p}\Gamma(e_j+\gamma_j k)}{\prod_{j=1}^{q}\Gamma(f_j+\delta_j k)}\frac{a^k x^{\zeta k}(t-x)^{\eta k+u}}{k!}\frac{z^u}{u!}$$

$$\times\frac{1}{2\pi i}\int_L \varphi(\xi)\,y^{\xi}x^{\mu\xi}(t-x)^{\nu\xi}\,d\xi\,dx$$

which in view of (4), becomes

$$I_1 = e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{u}\frac{\prod_{j=1}^{p}\Gamma(e_j+\gamma_j k)}{\prod_{j=1}^{q}\Gamma(f_j+\delta_j k)}\times\frac{a^k x^{\zeta k}(t-x)^{\eta k+u-k}}{k!}\times\frac{z^{u-k}}{(u-k)!}$$

$$\times\frac{1}{2\pi i}\int_L \varphi(\xi)y^{\xi}x^{\mu\xi}(t-x)^{\nu\xi}d\xi dx$$

Interchanging the order of integration and summation, we obtain

$$I_1 = e^{-zt}\sum_{u=0}^{\infty}\sum_{k=0}^{u}f(k)\frac{z^{u-k}}{(u-k)!}\times\frac{1}{2\pi i}\int_L \varphi(\xi)y^{\xi}\left\{\int_0^t x^{\rho+\zeta k+\mu\xi-1}(t-x)^{\sigma+(\eta-1)k+u+\nu\xi-1}dx\right\}d\xi$$

where $f(k)$ is given by (6).

Further on substituting $x = ts$ in the inner x- integral, the above expression reduces to

$$I_1 = e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{u}f(k)\frac{z^{u-k}}{(u-k)!}t^{(\gamma+\eta-1)k+u}\times\frac{1}{2\pi i}\int_L \varphi(\xi)y^{\xi}\left\{\int_0^1 s^{\rho+\zeta k+\mu\xi-1}(1-s)^{\sigma+(\eta-1)k+u+\nu\xi-1}ds\right\}d\xi$$

$$= e^{-zt} t^{\rho+\sigma-1} \sum_{u=0}^{\infty} \sum_{k=0}^{u} f(k) \frac{z^{u-k}}{(u-k)!} t^{(\gamma+\eta-1)k+u} \times \frac{1}{2\pi i} \int_L \varphi(\xi) \frac{\Gamma(\rho+\zeta k+\mu\xi)\Gamma(\sigma+(\eta-1)k+u+\nu\xi)}{\Gamma(\rho+\sigma+(\zeta+\eta-1)k+u+(\mu+\nu)\xi)} y^{\xi} t^{(\mu+\nu)\xi} d\xi$$

Finally, by virtue of (1), we arrive at the desired result.

Second Integral

$$I_2 \equiv \int_0^t x^{\rho-1}(t-x)^{\sigma-1} e^{-xz} {}_p\psi_q \left[\begin{matrix} (e_j,\gamma_j)_{1,p}; \\ (f_j,\delta_j)_{1,q}; \end{matrix} ax^{\zeta}(t-x)^{\eta} \right]$$

$$\times \; I^{m,n}_{p_i,q_i:r} \left[yx^{-\mu}(t-x)^{-\nu} \left| \begin{matrix} (a_j,\alpha_j)_{1,n}; (a_j,\alpha_j)_{n+1,p_i} \\ (b_j,\beta_j)_{1,m}; (b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right. \right] dx$$

$$= e^{-zt} t^{\rho+\sigma-1} \sum_{u=0}^{\infty} \sum_{k=0}^{u} f(k) \frac{z^{u-k}}{(u-k)!} t^{(\zeta+\eta-1)k+u}$$

$$\times I^{m,n+2}_{p_i+2,q_i+1:r} \left\{ yt^{-\mu-\nu} \left| \begin{matrix} (a_j,\alpha_j)_{1,n}; (a_{ji},\alpha_{ji})_{n+1,p_i}, (\rho+\sigma+(\zeta+n-1)k+u,\mu+\nu) \\ (\rho+\zeta k,\mu), (\sigma+(\eta-1)k+u,\nu), (b_j,\beta_j)_{1,m}; (b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right. \right\} \quad (7)$$

Provided $\mathrm{Re}(\rho) - \mu \max_{1\le j\le n} [\mathrm{Re}(\frac{a_j-1}{\alpha_j})] > 0$, $\mathrm{Re}(\sigma) - \nu \max_{1\le j\le n} [\mathrm{Re}(\frac{a_j-1}{\alpha_j})] > 0$,

along with the sets of conditions (i) to (iii) given with I_1 and $f(k)$ is given by (6).

Proof: It can be proved on lines similar to those of (5).

Third Integral

$$I_3 = \int_0^t x^{\rho-1}(t-x)^{\sigma-1} e^{-xz} {}_p\psi_q \left[\begin{matrix} (e_j,\gamma_j)_{1,p}; \\ (f_j,\delta_j)_{1,q}; \end{matrix} ax^{\zeta}(t-x)^{\eta} \right]$$

$$\times I^{m,n}_{p_i,q_i:r} \left\{ yx^{\mu}(t-x)^{-\nu} \left| \begin{matrix} (a_j,\alpha_j)_{1,n}; (a_{ji},\alpha_{ji})_{n+1,p_i} \\ (b_j,\beta_j)_{1,m}; (b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right. \right\} dx$$

$$= e^{-zt} t^{\rho+\sigma-1} \sum_{u=0}^{\infty} \sum_{k=0}^{u} f(k) \frac{z^{u-k}}{(u-k)!} t^{(\zeta+\eta-1)k+u}$$

$$\times I^{m+1,n+1}_{p_i+1,q_i+2:r} \left\{ yt^{\mu-\nu} \left| \begin{matrix} (1-\rho-\zeta k,\mu), (a_j,\alpha_j)_{1,n}; (a_{ji},\alpha_{ji})_{n+1,p_i} \\ (\sigma+(\eta-1)k+u,\nu), (b_j,\beta_j)_{1,m}; (b_{ji},\beta_{ji})_{m+1,q_i}, (1-\rho-\sigma-(\zeta+\eta-1)k-u,\mu-\nu) \end{matrix} \right. \right\} \quad (8)$$

Provided $\mathrm{Re}(\rho) + \mu \min_{1\le j\le m} [\mathrm{Re}(\frac{b_j}{\beta_j})] > 0$, $\mathrm{Re}(\sigma) - \nu \max_{1\le j\le n} [\mathrm{Re}(\frac{a_j-1}{\alpha_j})] > 0$, along with the sets of conditions (i) to (iii) given with I_1 and $f(k)$ is given by (6)

Proof: It can be proved on lines similar to those of (5).

Fourth Integral

$$I_4 \equiv \int_0^t x^{\rho-1}(t-x)^{\sigma-1} e^{-xz} \; {}_p\psi_q\left[\begin{matrix}(e_j,\gamma_j)_{1,p};\\ (f_j,\delta_j)_{1,q};\end{matrix}\; ax^{\zeta}(t-x)^{\eta}\right]$$

$$\times I^{m,n}_{p_i,q_i:r}\left[yx^{-\mu}(t-x)^{\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_j,\alpha_j)_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right] dx$$

$$= e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{u} f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I^{m+1,n+1}_{p_i+2,q_i+1:r}\left\{yt^{-\mu+\nu}\left|\begin{matrix}(1-\sigma-(\eta-1)k-u,\nu),(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},(\rho+\sigma+(\zeta+\eta-1)k+u,\mu-\nu)\\ (\rho+\zeta k,\mu),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\} \quad (9)$$

provided $\mathrm{Re}(\rho)+\mu \min\limits_{1\le j\le m}[\mathrm{Re}(b_j/\beta_j)]>0$, $\mathrm{Re}(\sigma)-\nu \max\limits_{1\le j\le n}[\mathrm{Re}(\frac{a_j-1}{\alpha_j})]>0$, along with the sets of conditions (i) to (iii) given with I_1 and $f(k)$ is given by (6).

Proof: It can be proved on lines similar to those of (5).

3. PARTICULAR CASES

- On taking $\gamma_j = 1 = \delta_j$ in equation (5), (7), (8), and (9), we get the following four integrals respectively, involving generalized hypergeometric function

$$(1)\quad \int_0^t x^{\rho-1}(t-x)^{\sigma-1} e^{-xz} \; {}_pF_q\left[\begin{matrix}(e_j)_{1,p};\\ (f_j)_{1,q};\end{matrix}\; ax^{\zeta}(t-x)^{\eta}\right]$$

$$\times\; I^{m,n}_{p_i,q_i:r}\left[yx^{\mu}(t-x)^{\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_j,\alpha_j)_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right] dx$$

$$= e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{u} g(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I^{m,n+2}_{p_i+2,q_i+1:r}\left\{yt^{\mu+\nu}\left|\begin{matrix}(1-\rho-\zeta k,\mu),(1-\sigma-(n-1)k-u,\nu)(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i},(1-\rho-\sigma-(\zeta+\eta-1)k-u,\mu+\nu)\end{matrix}\right.\right\} \quad (10)$$

$$g(k) = \frac{\prod_{j=1}^{p} \Gamma(e_j + k)\Gamma(e_j)}{\prod_{j=1}^{q} \Gamma(f_j + k)\Gamma(f_j)} \frac{a^k}{k!}$$

Where (11)

along with the sets of conditions (i) to (iv) given with I_1 .

(2) $$\int_0^t x^{\rho-1}(t-x)^{\sigma-1} e^{-xz} \; {}_pF_q\left[\begin{matrix}(e_j)_{1,p};\\ (f_j)_{1,q};\end{matrix}\; ax^{\zeta}(t-x)^{\eta}\right]$$

$$\times \; I^{m,n}_{p_i,q_i:r}\left[yx^{-\mu}(t-x)^{-\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_j,\alpha_j)_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right] dx$$

$$= e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{u} g(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I^{m,n+2}_{p_i+2,q_i+1:r}\left\{yt^{-\mu-\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},(\rho+\sigma+(\zeta+n-1)k+u,\mu+\nu)\\ (\rho+\zeta k,\mu),(\sigma+(\eta-1)k+u,\nu),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\} \quad (12)$$

along with the sets of conditions (i) to (iv) given with I_2 and $g(k)$ is given by (11).

(3) $$\int_0^t x^{\rho-1}(t-x)^{\sigma-1} e^{-xz} \; {}_pF_q\left[\begin{matrix}(e_j)_{1,p};\\ (f_j)_{1,q};\end{matrix}\; ax^{\zeta}(t-x)^{\eta}\right]$$

$$\times \; I^{m,n}_{p_i,q_i:r}\left[yx^{\mu}(t-x)^{-\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_j,\alpha_j)_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right] dx$$

$$= e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{u} g(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I^{m+1,n+1}_{p_i+1,q_i+2:r}\left\{yt^{\mu-\nu}\left|\begin{matrix}(1-\rho-\zeta k,\mu),(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (\sigma+(\eta-1)k+u,\nu),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i},(1-\rho-\sigma-(\zeta+\eta-1)k-u,\mu-\nu)\end{matrix}\right.\right\}$$

along with the sets of conditions (i) to (iv) given with I_3 and $g(k)$ is given by (11).

$$(4)\quad \int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-xz}\ {}_pF_q\left[\begin{matrix}(e_j)_{1,p};\\(f_j)_{1,q};\end{matrix}\ a\,x^{\zeta}(t-x)^{\eta}\right]$$

$$\times I^{m,n}_{p_i,q_i:r}\left[y x^{-\mu}(t-x)^{\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_j,\alpha_j)_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dx$$

$$= e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{u} g(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I^{m+1,n+1}_{p_i+2,q_i+1:r}\left\{yt^{-\mu+\nu}\left|\begin{matrix}(1-\sigma-(\eta-1)k-u,\nu),(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i},(\rho+\sigma+(\zeta+\eta-1)k+u,\mu-\nu)\\(\rho+\zeta k,\mu),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right\}$$

along with the sets of conditions (i) to (iv) given with I_4 and $g(k)$ is given by (11).

- Further, on specifying the parameters suitably, the generalized hypergeometric function reduces to Gauss hypergeometric function and we get the results due to Saha et al. [5].
- On taking t = 1, η = 0 and r = 1, I function reduces to Fox's H- function ([6], [7] and [8]) and we get the results due to Arora and Saha [9].

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REFERENCES

1. V. P. Saxena, The I-Function, Anamaya Publishers, New Delhi, (2008).
2. E. M. Wright, *The asymptotic expansion of the generalized hypergeometric function*, Proc. London Math. Soc. **46** (1946), 389–408.
3. H. M. Srivastava, H. L. Manocha, *A Treatise on Generating Functions*, Halsted Press (Ellis Horwood Limited, Chichester) 1984.
4. E. D. Rainville, Special Functions, Chelsea Publication Company, Bronx, New York, (1971).
5. U. K. Saha, L. K. Arora and B. K. Dutta, Integrals Involving I-Function, Gen. Math. Notes, 6(1), (2011), 1-14.
6. C. Fox, The G- and H-functions as symmetrical Fourier kernels, Trans. Amer. Math. Soc., 98(1961), 395-429.

7. A. M. Mathai and R.K. Saxena, The H-Function with Applications in Statistics and Other Disciplines, Wiley Eastern Limited, New Delhi, Bangalore, Bombay, (1978).

8. H. M. Srivastava, K. C. Gupta and S. P. Goyal, The H-Functions of One and Two Variables with Applications, South Asian Publishers, New Delhi, Madras, (1982).

9. L. K. Arora and U. K. Saha, Integrals involving hypergeometric function and H-function, J. Indian Acad. Math., 32(1) (2010), 243-249.